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Using CAD parameter sensitivities for stack-up tolerance allocation

Keywords: Tolerance allocation, Tolerance analysis, Computer-Aided Design, Parametric Modelling

Abstract

Tolerance allocation is an important step in the design process. It is necessary to produce high quality components cost effectively. However, the process of allocating tolerances can be time consuming and difficult, especially for complex models. This work demonstrates a novel CAD based approach, where the sensitivities of product dimensions to changes in the values of the feature parameters in the CAD model are computed. These are used to automatically establish the assembly response function for the product. This information has been used to automatically allocate tolerances to individual part dimensions to achieve specified tolerances on the assembly dimensions, even for tolerance allocation in more than one direction simultaneously. It is also shown how pre-existing constraints on some of the part dimensions can be represented and how situations can be identified where the required tolerance allocation is not achievable. A methodology is also presented that uses the same information to model a component with different amounts of dimensional variation to simulate the effects of tolerance stack-up.

1 Introduction

To achieve product quality, the amount of variation in the product has to be maintained within limits. Product quality is generally characterized by a group of features that affect the design functionality and the level of customer satisfaction. The control of the variation in this group of features is achieved through the tolerance allocation. The aim of tolerance allocation is to optimally allocate tolerances to the dimensions of the features which are manufactured, to achieve a suitably small variation in the dimensions which impact quality and performance [1]. Optimally is usually understood to mean with respect to some measure of manufacturing cost, which increases with tighter tolerances.

Tolerances are usually divided into dimensional tolerances, which limit the variation of the dimensions of the features, and geometric tolerances which limit the variation of the geometric shape. Both types of tolerance allocation are active research areas. Examples of previous research in dimensional tolerancing is summarised in [2], [3], [4]. Examples of research into geometrical tolerance allocation include [5], [6], [7].

Herein the focus is dimensional tolerance allocation. The terminology of Key Characteristics [1] has been adopted for describing the different dimensions to which tolerances are applied. In this work Key Product Characteristic (KPC) dimensions are specified to achieve product quality. Key Control Characteristic (KCC) dimensions are the individual component dimensions which are manufactured and which have an effect on the KPCs. In a product it is assumed that the Key Product Characteristic dimensions are assembly dimensions which are not directly manufactured, and are a function of the constituent component's Key Control Characteristic dimensions. For example, in Figure 1 the distance between the holes in Part 2 and Part 3 significantly affect the performance of the product. As such dimension z_1 is a KPC. The KCCs are x_1 , x_2 , and x_3 , as these belong to the components which are actually manufactured and have an effect on the KPCs.

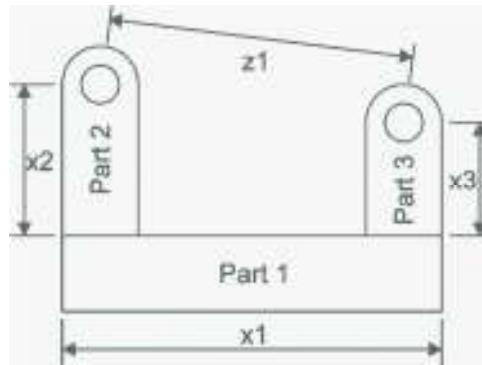


Figure 1 - KPC and KCC dimensions in an assembly

A systematic approach to tolerance design involves iterative computation following two alternate approaches, tolerance analysis and tolerance synthesis (allocation) [8]. In tolerance analysis the KPC tolerance is unknown. It is calculated by applying tolerances to KCCs based on standards, regulations and previous know-how, and then determining the overall tolerance for the KPC based on the cumulative effect on the KCCs. High and low limits are then calculated for the KPC. In tolerance allocation the KPC tolerance is specified as a design requirement. Tolerances are then allocated to the KCC dimensions such that the required dimensional variation on the KPC is achieved, considering the fact that the tolerances on the individual KCCs will stack-up. Singh has provided papers summarising research on each topic [8], [9].

Herein, a process for tolerance allocation is described. The aim is to allocate tolerances to the KCC dimensions to achieve a suitably small variation in the KPCs, to ensure target performance [1]. For example, for the product in Figure 1, tolerance allocation must calculate sufficiently small limits of variation in the lengths of x_1 , x_2 , and x_3 in order to limit the variation in the dimension z_1 when the parts are assembled. Allocated tolerances are usually defined in terms of the higher and lower limits for a dimension which are deemed to be acceptable.

Within the modern design process virtual geometric representations of the parts which make up a product are usually created in a 3D feature based CAD system and assembled together. Tolerance allocation within CAD modelling has been investigated by [10], [11], [12], [13], [14], [15] but the typical route to tolerance allocation is through the use of graphs or charts. In current commercial CAD systems tolerance information is still represented in the form of notes, symbols, and labels similar to what is normally found on engineering drawings, as the outcome of manual interaction [16], [17], [18], [19].

In most major CAD systems the shape and size of the part models are defined using features and parameters. If any part dimensions need to be altered the values of the parameters which define the CAD models are changed. As such, assembly dimensions are functions of individual CAD feature parameters and any variation in the feature parameters can directly affect the assembly dimensions. Where the assembly dimension is a KPC, this change will also affect the performance of the assembly. In a parametric CAD system the designer has to model the shape of a part or assembly only once and can derive variants by changing parameter values. The parameters chosen to model a part may or may not directly represent a dimension in the manufactured part to which tolerances are allocated. It is therefore not always clear what effect the feature parameters have on the KCC and KPC values.

Herein, a generic, CAD based method for tolerance allocation is presented. It uses sensitivity information calculated for the parameters in the CAD model to automatically derive the relationship between the KCC and KPC dimensions, which is also known as the assembly response function. This information is then used to allocate tolerances to the KCCs to achieve the desired tolerance for the KPC. Other research which has made use of sensitivities to parametric variations for tolerancing includes [20], [21]. The work supports an interactive design approach which allows the designer to predict tolerance values for a design. Due to the simplicity of the approach it can be used in the preliminary design stages, thereby allowing the

designer to assess from early on the suitability of the design for different manufacturing processes. The speed of the process means that the designer can quickly iterate through different configurations, interactively testing the suitability of each.

This approach provides a simple “worst-case” approach to tolerance allocation, where tolerances are distributed amongst the KCC dimensions based on the magnitude of the effect they have on the KPCs. The approach can account for situations where the designer does not have control over the tolerances which are applied to a KCC (e.g. when a bought in component with defined tolerance is included in the assembly). While this method of tolerance allocation does not account for other considerations, such as manufacturing cost, the fact it is available within the CAD environment will allow tolerances to be calculated quickly and easily. This will have utility for allowing tolerances to be considered earlier in the design process, or for providing an estimate of the individual tolerances from which to optimise the tolerance distribution. This facilitates integrated engineering, which involves taking into account all constraints in the product lifecycle early in the design process [22]. Having access to this information earlier in the design process will change the manner in which the designer interacts with the design and is one route to the ambition of considering manufacture earlier in the design process. It will also allow feasibility assessments, and manufacture and assembly decisions to be made earlier.

This work aims to enhance the area of interactive design. Some obvious contributions are that [22] proposed an approach for functional tolerance allocation for the purposes of making manufacturing/costing trade-off decisions earlier in the design process. The focus was on optimizing the tolerance allocation and analysing the impact on manufacturing cost of the design choices made. As such, that work could be fed the information from the work herein. [23] considers an approach for integrating thermo-mechanical strains into the tolerance analysis. One of the requirements for this is an understanding of the three dimensional tolerance chains, which the methodology outlined in this paper can provide directly from the CAD parameterisation. In the work on integrated product–process design, [24], it is highlighted that it is the designer and/or process engineer who is responsible for eliminating tight fits through tolerancing. The work described here provides the designer with the information to evolve the design more easily and quickly than before. [14] and [15] describe approaches for interactively considering tolerances based on graphs which the work herein complements.

In section 2 the sensitivity of the parameters defining the CAD model to dimensions in the part and assembly models is used to establish a link between the KCC and KPC dimensions. In section 3 these relationships are used to allocate tolerances in simple stack-up type assemblies. In section 4 the information is used to simulate the effect of dimensional variation in a CAD model, allowing the stack-up effects of the variation in individual component dimensions to be determined. Sections 5 and 6 discuss and conclude the work.

2 Using CAD parameters to automatically establish the relationship between the KPC and KCC dimensions in a CAD model assembly

In this work an assembly dimension refers to a dimension that is not directly manufactured, but exists between features of different components when assembly has taken place. In this work assembly dimensions which have an effect on product quality are referred to as KPCs. They are a function of some of the dimensions of the components of which the assembly is comprised, which are referred to as KCCs.

In this section the link between the KPC and the KCC dimensions is computed automatically using the CAD system. This relationship, also known as the assembly response function, is necessary to be able to allocate tolerances on the KCC dimensions in order to achieve specified tolerances on the KPC dimensions.

2.1 Setting up the CAD model

This work was implemented such that no constraints were imposed on the way in which the model was parameterised, other than that the features in the model had to be fully defined. This is good practice which is to be expected. This work was implemented using the Visual Basic API for CATIA V5. Within the CAD model the “measures” function was placed on each dimension representing a KPC or KCC in the model. This creates a sensor which reports the value of the measurement of interest, for the current state of the CAD model, but does not impose any constraint on it. As such, it reflects when a variation of the CAD parameters causes the KPC or KCC dimensions to change. All the CAD systems which the authors are familiar with offer similar functionality, although the terminology used to describe it may differ. In most systems the measured values can appear on the feature tree and can also be accessed through the modeller API.

2.2 Perturbing the CAD model parameters

To determine the link between the CAD feature parameters and the KPC and KCC dimensions, a finite difference approach is used. Using the scripting interface to the CAD system, each feature parameter in the model is perturbed by a small amount, ΔP , in turn. The effect of the feature parameter perturbation on each KPC and KCC dimension is recorded.

Even CAD models of simple components may contain a large number of parameters. Many parameters may not have been explicitly created by the user, but are created by the CAD software in the background in order to define the features being created [25]. The goal of this process is to calculate the tolerances automatically without imposing any constraints on the features used to build the model.

Not all of the parameters in the model are suitable for perturbation in the manner described, and so filtering of the parameters is required. For the described approach a suitable parameter to be perturbed must have a real value and represent a length or angle parameter. Small perturbations of these parameters typically result in correspondingly small changes in model shape, and therefore small changes in measured dimension. In order to be considered a suitable parameter for perturbation, a parameter must meet the following criteria:

- It must allow read/write access.
- It must not be defined as a function of other parameters. If it is then the perturbation of the other parameters will yield appropriate sensitivity values.
- It must not be an integer, Boolean or string value. Integer values (such as number of features in a pattern) and Boolean values (such as the activity of a feature) typically cause significant changes in the shape of the model, and for this reason are not suitable for this type of analysis. Perturbation of string parameters (such as names and labels) is essentially meaningless in this context and will not result in a shape change.

Where a feature parameter is identified as suitable for evaluation, based on the criteria above, its value attribute is perturbed by a small amount, ΔP and the model is updated. At this point the change in each of the measured dimensions is calculated and recorded. The parameter value is then reset to its original value before the value of the next suitable parameter is identified and perturbed.

The process has been implemented using a ΔP which is small compared to the value of the parameter.

2.3 Calculating the sensitivities for the KPCs and KCCs to model parameters

For each parameter, P , the sensitivity, S , is computed as the change in each measured dimension, ΔKC , to the small change applied to the value of each feature parameter, ΔP . The relationship between the change in CAD parameter value, change in measured dimension and the sensitivity is

$$S = \frac{\Delta KC}{\Delta P}, \quad (1)$$

where KC represents the measurement of either a KPC or a KCC.

The sensitivity value for each KC dimension and CAD parameter combination is recorded in the sensitivity matrix for the model. Each KC dimension will be represented by a row in the sensitivity matrix. For each feature parameter which is perturbed, a column will exist in the sensitivity matrix. Each entry in the matrix represents the sensitivity of change in the value of the KC dimension represented by the row due to a perturbation of the feature parameter represented by the column. For example in a model with two parameters and two Key Characteristic dimensions, the sensitivity matrix will have the form

$$\begin{Bmatrix} \Delta KC1 \\ \Delta KC2 \end{Bmatrix} = \begin{bmatrix} \frac{\Delta KC1}{\Delta P1} & \frac{\Delta KC1}{\Delta P2} \\ \frac{\Delta KC2}{\Delta P1} & \frac{\Delta KC2}{\Delta P2} \end{bmatrix} \begin{Bmatrix} \Delta P1 \\ \Delta P2 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} \Delta P1 \\ \Delta P2 \end{Bmatrix}, \quad (2)$$

The sensitivity matrix is in effect a mapping between the parameters defining the CAD model and the KPC or KCC dimensions which have been specified.

2.4 Determining the relationship between the KCC and KPC dimensions

Separate sensitivity matrices can be calculated for the KPC and the KCC dimensions. The result is a sensitivity matrix, S_{cc} , relating the CAD feature parameters to the KCCs and a sensitivity matrix, S_{pc} , relating the CAD feature parameters to the KPCs. Therefore

$$\Delta KCC = [S_{cc}] \Delta P \quad (3)$$

and

$$\Delta KPC = [S_{pc}] \Delta P. \quad (4)$$

The predicted vector of parameter changes ΔP required to cause any desired change in the Key Control Characteristics ΔKCC can be computed using the pseudo-inverse of S_{cc} as

$$\Delta P = [S_{cc}]^+ \Delta KCC. \quad (5)$$

Note that as the inverse of a matrix is only achievable for a square, non-singular matrix, the more general solution used in this work is to use the pseudo inverse of the sensitivity matrix [26], which is denoted throughout by a superscript +, and can be computed using the Singular Value Decomposition. This pseudo-inverse and the inverse are the same where the inverse exists. If the inverse does not exist, Eqn (5) gives the vector of small parameter changes which give the best least squares approximation to the desired small changes in KCCs.

Substituting (5) into (4) gives an expression for the KPCs in terms of the KCCs as

$$\Delta KPC = [S_{pc}] [S_{cc}]^+ \Delta KCC = [\psi] \Delta KCC, \quad (6)$$

where ψ is a matrix relating the two, referred to here as the assembly response matrix. This expresses small changes in the KPCs in terms of small changes in the KCCs. This is the classic stack-up linear relationship, where the variation in a KPC is equal to the variation in each KCC dimension multiplied by a sensitivity.

Assuming the calculated sensitivities are constant for ΔKPC and ΔKCC in the range of the tolerance limits (which is reasonable because tolerances are typically very small compared to dimensional values), and that the model is fully defined (with all dimensions represented as KPCs and KCCs and no un-dimensioned gaps), then (6) can be expressed as the functional equation for the assembly within this small range, as

$$KPC = [\psi] KCC. \quad (7)$$

Note that from this point onward the Δ is assumed for the KPC and KCC notations. (7) approximates the derivative of each KPC relative to the KCCs in the model. This information is

a requirement for many tolerance based approaches where tolerance graphs, charts or chains have to be calculated. The ψ matrix makes it clear to the designer which KCC dimensions have a greatest effect on the KPCs. This information will allow the designer interact with the design in a manner that the likely impact of design iterations on the perceived quality of the resulting component is clear. In this work it imposes a requirement on the person defining the KCCs in the CAD model to ensure that all part dimensions which will have an effect on the KPCs have a “measure” item applied. The importance of understanding the variation effect of the KCCs on the KPCs is highlighted by [27].

2.5 Determining the relationship between the KCC and KPC dimensions - Example model 1

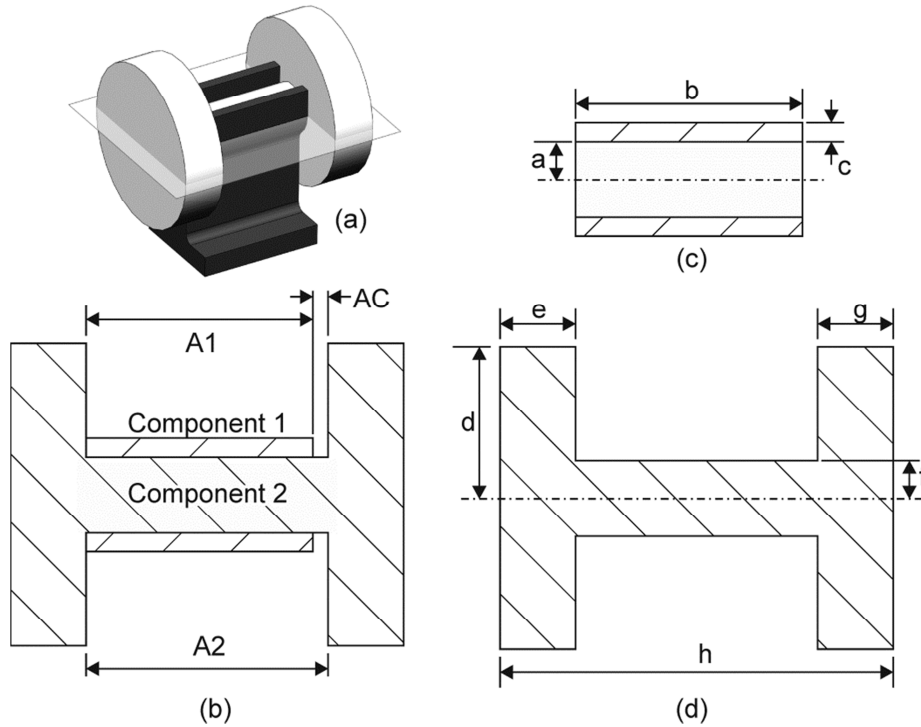


Figure 2 – Example model 1 – A two component assembly model (a) isometric view (b) assembly model with KCCs and KPC (c) component 1 CAD parameters (d) component 2 CAD parameters

Figure 2(a) shows an example product in which two components are assembled together. In order for the product to function well a specified axial clearance, AC, must exist between Component 1, the darker shaded housing, and Component 2, the lighter shaded roller component. For this example, the axial clearance is the KPC. A1 and A2 are manufactured dimensions which have an effect on the KPC and belong to Component 1 and Component 2 respectively. They are the KCCs in this example.

Figure 2(c) and Figure 2(d) show the feature parameters used to define Component 1 and Component 2 respectively. It should be noted that the KCC A2 is not directly represented by a CAD parameter in the component model.

For this example, computing the sensitivities allows (3) to be populated as

$$\begin{Bmatrix} \Delta A1 \\ \Delta A2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta b \\ \Delta c \\ \Delta d \\ \Delta e \\ \Delta f \\ \Delta g \\ \Delta h \end{Bmatrix} \quad (8)$$

and (4) to be populated to as

$$\{\Delta AC\} = \begin{bmatrix} 0 & -1 & 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta b \\ \Delta c \\ \Delta d \\ \Delta e \\ \Delta f \\ \Delta g \\ \Delta h \end{Bmatrix}. \quad (9)$$

Assuming that the calculated sensitivities are valid for all values of A1 and A2 (at least in the range of interest) and that the model is fully defined (with all dimensions represented as KPCs and KCCs and no un-dimensioned gaps), then from (7), the corresponding ψ matrix for this example is

$$[\psi] = \begin{bmatrix} -1 & +1 \end{bmatrix}, \quad (10)$$

which means that the relationship between the KPCs and KCCs can be written as

$$\mathbf{KPC} = \{AC\} = [\psi]\mathbf{KCC} = \begin{bmatrix} -1 & +1 \end{bmatrix} \begin{Bmatrix} A1 \\ A2 \end{Bmatrix}. \quad (11)$$

The functional equation which expresses AC in terms of A1 and A2 is therefore

$$AC = A2 - A1. \quad (12)$$

An analysis of the model in Figure 2 shows this relationship to be correct.

2.6 Determining the relationship between the KCC and KPC dimensions - Example model 2

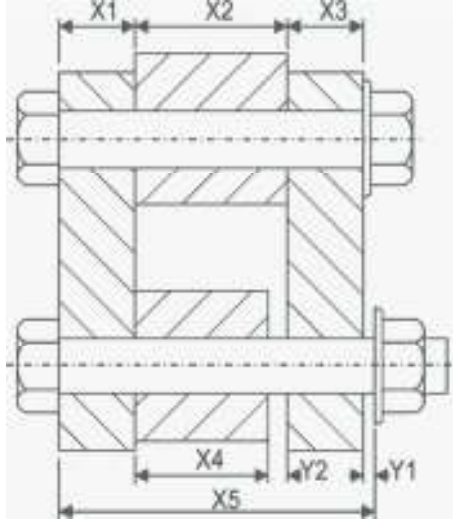


Figure 3 – Example model 2 - A wheel mounting assembly with two KPC values

A popular example from the literature is the wheel mounting assembly, approximated in Figure 3 [28]. This assembly has two clearances designated as KPC values (Y1 and Y2). For this assembly the relationship between the KPC and the KCC is derived to be

$$\{KPC\} = \begin{Bmatrix} Y1 \\ Y2 \end{Bmatrix} = [\psi]\{KCC\} = \begin{bmatrix} -1 & -1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \end{Bmatrix} \quad (13)$$

which yields the assembly response functions

$$\begin{aligned} Y1 &= -X1 - X2 - X3 + X5 \\ Y2 &= X2 - X4 \end{aligned} \quad (14)$$

This concurs with [28].

3 Allocating tolerances in simple stack-up scenarios

Section 2 demonstrated how the assembly response equation relating the values of the KCCs to the KPCs in the model could be automatically determined using sensitivity information for the parameters defining the features in a CAD model. This response function is necessary for many approaches to tolerance allocation or tolerance analysis. In this section the relationship is used to allocate tolerance limits to the KCC values to achieve a specified tolerance on the KPCs. The matrix based process has been implemented through the CAD modeller scripting interface such that it calculates tolerances automatically.

3.1 Tolerance allocation for products comprised of “unconstrained” components

Tolerances can be specified in terms of the higher and lower limits that are acceptable for a dimension. To calculate tolerances for the KPC and KCC values, the high and low limits for each have to be considered. To allow for this the KPC and KCC matrices in (7) are partitioned to account for the limits, giving KPC_p and KCC_p respectively in

$$[KPC_p] = [\psi][KCC_p]. \quad (15)$$

The KCC and KPC matrices are partitioned into two columns, one representing each limit. The order of the high and low limit terms in each row is dictated by the relationship between the KCCs and the KPCs. Herein the convention has been adopted that, in the absence of other constraints, the left hand column is the high limit for the KPC and the right hand column contains the low limit. Constraining factors may require the order of the limits in KPC_p to be reversed for models with more than one KPC. For example, when the order of the high and low limit for a KCC is determined based on one KPC, this will act as a constraint on the order of the limits for other KPCs affected by the same KCC. An example of this is shown in section 3.1.2.

The KCC matrix is also partitioned, for which the rule is applied that:

- i. if the corresponding term in the ψ matrix is positive for a given KPC and KCC combination, then the order of the high and low limits for that row in the KCC matrix is the same as for the KPC in the KPC_p matrix.
- ii. if the corresponding term in the ψ matrix is negative for a given KPC and KCC combination, then the order of the high and low limits for that row in the KCC matrix should be the reverse of those for the KPC in the KPC_p matrix.

The process for defining the order of the limits in the partitioned matrices means it is possible to identify that the process is over constrained. This will be due to the fact that there are many KPCs to which tolerances have to be allocated, which are all dependent on the same group of KCC dimensions. Being able to identify this fact automatically is valuable.

Once the matrix has been partitioned in this way, the limits of the KCC values to achieve a specified tolerance limit allocation on the KPC values can be calculated as

$$[KCC_p] = [\psi]^+ [KPC_p]. \quad (16)$$

3.1.1 Tolerance allocation for Example model 1 - all components “unconstrained”

With reference to the model shown in Figure 2, the partitioned matrices are

$$[KPC_p] = [AC^H \quad AC^L] \text{ and } [KCC_p] = \begin{bmatrix} A1^L & A1^H \\ A2^H & A2^L \end{bmatrix}, \quad (17)$$

where the KCC matrix is partitioned as described above, based on the ψ matrix in (10).

Should the specified axial clearance for the model be

$$AC = 0_{+0.05}^{+0.15}, \quad (18)$$

and given that for this model

$$\psi^+ = \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix}, \quad (19)$$

then the limits on the KCCs are calculated from (16) to be

$$[KCC_p] = \begin{bmatrix} A1^L & A1^H \\ A2^H & A2^L \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix} \begin{bmatrix} 0.15 & 0.05 \end{bmatrix} = \begin{bmatrix} -0.075 & -0.025 \\ 0.075 & 0.025 \end{bmatrix}. \quad (20)$$

This method gives tolerance limits of $A1 \pm_{0.075}^{0.025}$ and $A2 \pm_{0.025}^{0.075}$. An analysis of the model shows this tolerance allocation to be appropriate.

3.1.2 Tolerance allocation for Example model 2 - all components “unconstrained”

For Example model 2, Figure 3, the ψ matrix in (13) has one column which has more than one non-zero value (column 2). This indicates a KCC has an effect on more than one KPC. Referring to the model in Figure 2 it is obvious that KCC X2 will have an effect on both Y1 and Y2, and will therefore constrain the order that their limits are defined in the partitioned matrices. As the sign of the value in the matrix is different for different rows, this indicates that the order of the high and low limits for both KPCs will have to be defined differently. So, if the KPC Y1 is specified to have its high limit in the left hand column and low limit in the right hand column (as is the convention), the KPC Y2 will have the opposite arrangement, giving

$$KPC_p = \begin{bmatrix} Y1^H & Y1^L \\ Y2^L & Y2^H \end{bmatrix}. \quad (21)$$

The KCC_p matrix therefore becomes

$$KCC_p = \begin{bmatrix} X1^L & X1^H \\ X2^L & X2^H \\ X3^L & X3^H \\ X4^H & X4^L \\ X5^H & X5^L \end{bmatrix}. \quad (22)$$

Note that in (13) the first row in ψ , which refers to Y1, the columns relating to X1, X2 and X3 contain negative values. The column representing X5 has a positive value. This has the effect that the high and low limits for X1 to X3 in KCC_p , (22) have the opposite order to Y1 in KPC_p , while for X5 the order is the same.

As the X2 order has been constrained when considering Y1, but it has an effect on Y2, this constrains the creation of the KPC_p matrix row relating to Y2. As the value in the ψ matrix is positive for the term corresponding to Y2 and X2, then the Y2 term must have the same order as X2. The X4 term is negative in ψ , meaning that in (22) the X4 row has a different high and low limit order than for the KPC Y2.

For this example, the pseudo-inverse of ψ is

$$\psi^+ = \begin{bmatrix} -2/7 & -1/7 \\ -1/7 & 3/7 \\ -2/7 & -1/7 \\ -1/7 & -4/7 \\ 2/7 & 1/7 \end{bmatrix}. \quad (23)$$

For a model where the limits on the KPCs are specified to be

$$Y1 \pm_{0.7}^{0.7} \text{ and } Y2 \pm_{0.7}^{0.7} \quad (24)$$

then $X1 \pm_{0.1}^{0.1}$, $X2 \pm_{0.4}^{0.4}$, $X3 \pm_{0.1}^{0.1}$, $X4 \pm_{0.3}^{0.3}$ and $X5 \pm_{0.1}^{0.1}$.

An analysis of the model shows this tolerance allocation to be appropriate.

3.2 Tolerance allocation for products which include “constrained” components

In section 3.1 the process of allocating tolerances to KCCs was described and demonstrated for models where all KCCs in the model were unconstrained. However, a product might include some components which are already in production and for which tolerance limits have already been specified. For example a bought in component might be included.

To account for this the ψ matrix and the KCC matrix are further partitioned into the terms which represent constrained KCCs (for which the designer has no control, denoted by subscript C) and unconstrained KCCs (where the designer has control, denoted by subscript U), giving

$$[KPC_P] = [\psi_U \quad \psi_C] \begin{bmatrix} KCC_{PU} \\ KCC_{PC} \end{bmatrix}. \quad (25)$$

In (25) KCC_{PU} is the terms in the KCC matrix representing unconstrained KCCs. KCC_{PC} represents the terms in the KCC matrix representing constrained KCCs. ψ_U are the terms in the ψ matrix representing the sensitivity of the KPC to the unconstrained KCCs. ψ_C are the terms in the ψ matrix representing the sensitivity of the KPC to the constrained KCCs.

Partitioning the matrix in this way, and substituting into (16), allows an expression to be derived for calculating the tolerances for the components being designed as

$$[KCC_{PU}] = [\psi_U]^+ ([KPC_P] - [\psi_C][KCC_{PC}]). \quad (26)$$

Note that where there are no constrained components in the assembly then (26) reduces to (16). This information allows the designer to interactively test how different bought in components would affect the perceived quality and manufacturing requirements of the design. The simplicity of the approach would allow the inclusion of these bought in components to be decided on earlier in the design process. This is information that would usually not be available until much later.

3.2.1 Tolerance allocation for Example model 1 – “constrained” and “unconstrained” components

Consider again the model in Figure 2(a). Should the specified axial clearance for the model be that shown in (18), and the limits for A2 be constrained to be

$$A2_{+0.02}^{+0.07}, \quad (27)$$

(26) can be used to allocate the tolerance limits on A1. For this example $\psi_C = [1]$, $\psi_U = [-1]$ and $KCC_{PU} = [A2^H \ A2^L] = [0.07 \ 0.02]$.

The result from inserting these values into (26) is

$$A1_{-0.08}^{-0.03}. \quad (28)$$

An analysis of the model shows this tolerance allocation to be appropriate.

3.2.2 Tolerance allocation for Example model 1 – “constrained” and “unconstrained” components, where tolerance allocation is not possible

Consider again the model in Figure 2(a). This time, should the specified axial clearance for the model be that shown in (18), and the limits for A2 be

$$A2_{+0.02}^{+0.18}, \quad (29)$$

then the result from inserting these values into (26) is

$$A1_{+0.03}^{-0.03}. \quad (30)$$

The fact that the lower limit calculated for A1 is greater than the higher limit indicates that tolerances cannot be successfully applied to A1 to provide the desired KPC variation. In this case a better quality constrained component (i.e. an A2 component with tighter tolerances) is

required, or the limits of variation for the KPC dimension must be slackened, while accepting the resulting reduction in product quality. With the use of this methodology, being able to identify situations such as this early in the design process is extremely useful, and could stop an infeasible design progressing.

3.2.3 Tolerance allocation for Example model 1 – “constrained” and “unconstrained” components

Considering again the model in Figure 3, where X4 belongs to a bought in component, and tolerances have to be allocated to dimensions X1, X2, X3 and X5 as KCCs.

For this example the limits on KPCs are specified as

$$Y1 = 0_{+0.1}^{+0.3} \text{ and } Y2 = 0_{+0.1}^{+0.2} \quad (31)$$

and the limits of the constrained dimension X4 are

$$X4_{0.0}^{0.05}. \quad (32)$$

For this model $\psi_c = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ and $\psi_U = \begin{bmatrix} -1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

Substituting these values into (26) gives

$$\begin{bmatrix} A1^L & A1^H \\ A2^L & A2^H \\ A3^L & A3^H \\ A5^L & A5^H \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} \\ 0 & 1 \\ -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \left(\begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \end{bmatrix} \begin{bmatrix} 0.05 & 0 \end{bmatrix} \right) = \begin{bmatrix} -0.15 & -0.1 \\ 0.15 & 0.2 \\ -0.15 & -0.1 \\ 0.15 & 0.1 \end{bmatrix}, \quad (33)$$

or

$$X1_{-0.15}^{-0.1}, X2_{0.15}^{0.2}, X3_{-0.15}^{-0.1} \text{ and } X5_{0.1}^{0.15}.$$

An analysis of the model shows this tolerance allocation to be appropriate.

3.2.4 Allocated tolerances in multiple directions

The approach presented has the advantage that it can be applied where tolerances are to be computed in multiple directions, whereas many other approaches can only be used to calculate tolerances in one direction at a time. Figure 4 shows an assembly model for a simplified belt tightener, the KCCs and KPCs for which are shown in Figure 5. Detail features in the model have been suppressed.

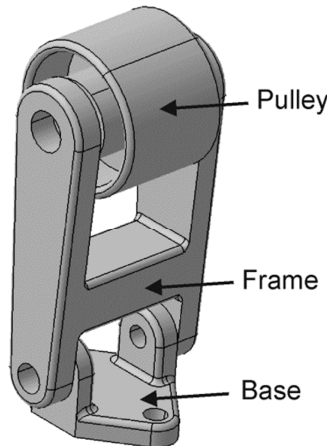


Figure 4 - Belt tightener assembly

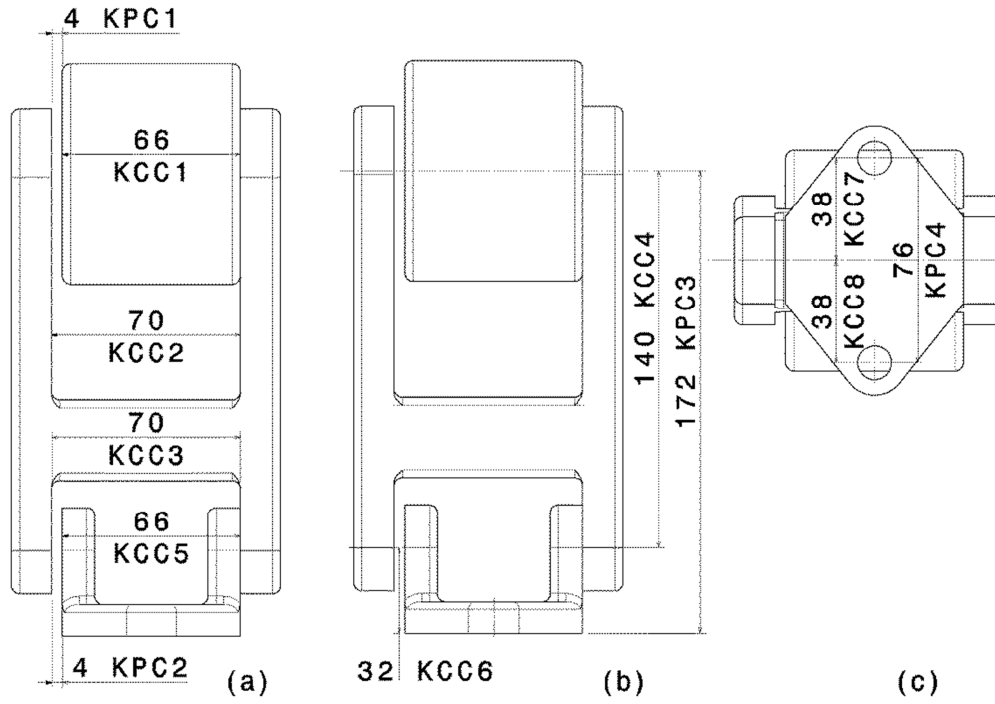


Figure 5 - Belt tightener assembly – KCCs and KPCs

In this assembly there are four KPCs. Two controlling axial clearances in one direction, one controls the spacing between the centre axes of the two rotation axes in another direction, and the final one specifies the spacing between the location holes and is specified in a direction perpendicular to the other KPC dimensions. When allocating tolerances to this model it can be assumed that the frame component is bought in, and as such the tolerances on it KCCs (KCC2, KCC3 and KCC4) are fixed.

If for this example the limits on KPCs are specified as

$$Y1 = 0^{+0.8}_{+0.3}, Y2 = 0^{+0.8}_{+0.3}, Y3 = 0^{-0.4}_{-0.8}, Y4 = 0^{+0.6}_{-0.2}, \quad (34)$$

and the limits of the dimensions defining the bought in frame component are

$$X2 = 0^{0.175}_{0.025}, X3 = 0^{0.175}_{0.025}, X4 = 0^{-0.1}_{-0.325}. \quad (35)$$

The tolerances for the remaining dimensions are calculated as

$$X1 = 0^{-0.025}_{-0.175}, X5 = 0^{-0.025}_{-0.175}, X6 = 0^{-0.1}_{-0.325}, X7 = 0^{0.225}_{-0.150}, X8 = 0^{0.225}_{-0.150}. \quad (36)$$

Again, an analysis of the model shows this tolerance allocation to be appropriate.

4 Simulating tolerance stack-up in assemblies

In Section 2, (3) gives the relationship between the KCC dimensions and the CAD model feature parameters. In this section this information is used to predict the perturbations to be applied to each CAD model feature parameter to achieve a given variation in each KCC dimension. This allows KCC variation to be simulated and once implemented the corresponding change in the KPC dimensions due to the stack-up effects can be determined by a simple measurement. Having a clearly defined link between the model parameterisation and the KCCs will allow the designer to interact with the model in a much more knowledgeable manner, with a better understanding about the likely consequences of design changes.

(5) is the vector of parameter changes caused by moving in the direction defined by the sensitivities in S_{cc} , computed using the pseudo-inverse of S_{cc} . Providing the variation to be simulated in each KCC in (5), and calculating the S matrix using the procedures in section 2, the

perturbations to be applied to the CAD feature parameters to cause the KCCs to change by the desired amount can be predicted.

For Example model 1, Figure 2, should there be a desire to simulate a change in length of A1 by -0.1mm, and a change in length of A2 by 0.2mm, then substituting the information into (5) gives

$$\begin{pmatrix} \Delta a \\ \Delta b \\ \Delta c \\ \Delta d \\ \Delta e \\ \Delta f \\ \Delta g \\ \Delta h \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix}^+ \begin{Bmatrix} -0.1 \\ 0.2 \end{Bmatrix} = \begin{pmatrix} 0 \\ -0.1 \\ 0 \\ 0 \\ -0.067 \\ 0 \\ -0.067 \\ 0.067 \end{pmatrix} \quad (37)$$

It can be observed that applying these parameter changes to the model in Figure 2(c) and (d) will result in the desired change in A1 and A2. Within the CAD model, using the “measures” sensor attached to the dimension AC, the resulting variation in the KPC can be measured. In this case the change in KPC dimension will be 0.3mm. This is the stack-up effect of the variation in each of the KCCs provided. The same process has been used to model the variation in KPCs due to a variation in KCCs in multiple directions, for example the model in Figure 5. Due to the nature of the approach three dimensional tolerance analysis in 3D CAD assemblies requires no additional effort.

While the same value for the variation in the KPC could be calculated using (6), being able to simulate the variation in the CAD environment is useful for a number of reasons. The designer can simulate different tolerance scenarios by setting the CAD model to represent different variations in dimensions. This will allow the designer to interact with the design at different variations, to carry out analysis of the design in this form. One example might be to utilise the CAD modeller interference tools to ensure that a given variation does not cause an unexpected interference to occur. Should interference occur, an approach to eliminating it is provided by [29]. It might be of benefit to be able to use the CAD modeller to measure the mass of the assembly when the parts are in their Maximal Material Condition (MMC), or to be able to subject the Least Material Condition (LMC) part models to a stress analysis to insure the required structural performance is still realised. Currently it is common for analysis such as this to only be carried out for a model in its nominal state, ignoring dimensional variations. Having access to this information, and early in the design process, is useful.

5 Discussion

Given a parametric CAD model with a set of measured dimensions defined, this work describes an automated approach for allocating tolerance limits to the measured dimensions. It uses a sensitivity analysis, relating small parametric perturbations to small measured dimension variations, to predict the relationship between the dimensions to which tolerances are to be applied. As such it allows tolerance allocation to be carried out at low cost, and earlier in the design process than it currently is. This will change the manner in which the designer interacts with the design, as it will be possible to consider tolerances and their effect on manufacturing, costing and quality much earlier in the design process. This means that much more information is available than before when determining how to progress a design.

It was shown in section 2 how the relationship calculated between the CAD parameters and the measured dimensions can also be used to compute a sensitivity matrix relating the KCCs and the KPCs. This information provides the assembly response function. It is a simpler method of computing it compared to the dimensional loop equation proposed in the Direct Linearization Method (DLM) [30] where each vector in the tolerance chain is formulated as a component dimension.

In section 3 a process is described to automatically allocate tolerances for the components in an assembly by linking them to the values of the feature parameters (i.e. KCCs and KPCs), which

defines the shape of the component models in the CAD assembly. When the tolerance limits for the KPC are specified as a design requirement, the sensitivity method presented has the ability to predict the tolerance limits allocated to the Key Control Characteristic dimensions to achieve it. The approach has been used to allocate tolerances in multiple directions simultaneously, which is of significant benefit and something that many existing tolerance allocation approaches do not offer.

The finite difference method employed in this work is straightforward and easily implemented. Calculating the sensitivities using the finite difference approach assumes a linearized relationship between the CAD feature parameters and the KCCs and KPCs. The assembly examples detailed in this paper have mostly had linear relationships between the feature parameters, the KCCs and the KPCs. The result is that the approach has been able to calculate accurate sensitivities between the KCC and the KPC to the CAD parameters, and therefore to each other. This has resulted in exact and correct tolerance limits being calculated for the example models. This also desensitised the approach to the size of ΔP chosen, as where there is a linear relationship the sensitivity will be the same for all perturbation sizes.

Where a non-linear relationship exists between a KCC, KPC or the CAD parameters then exact limits may not be provided, and the choice of ΔP used to calculate the sensitivities will have to be sufficiently small that the result from the pseudo-inverse calculation is sufficiently accurate. In the examples included within, a perturbation of 1mm was applied to the parameter values, as this is small compared to the magnitude of the dimensions being perturbed. It was also used as it made the description of the process easier. Where the designer is aware of the size of tolerances which can be expected from the model then to apply a perturbation of the order of the tolerance size is a valid approach. If this approach is to be used early in the design process to verify the feasibility of a design then approximate results may be acceptable. Where accuracy is a concern the calculated tolerances can be verified using existing approaches [31], or an approach derived from [14]. Also, as has already been commented, the simplicity of this approach allows for approximate tolerance allocations to be made which can then be optimised using other approaches [32], [33], [34], [35].

(20), (24) and (33) demonstrate one of the properties of the method, which is that it allocates tolerances to dimensions based on the sensitivity of the KCCs to the KPCs. In many stack-up type scenarios this will result in the same tolerance allocation being assigned to many of the KCC dimensions in a model.

The choice of KCCs is critical to the success of the approach, both in terms of achieving an accurate tolerance allocation and the ability to make the component cost effectively. Where a model has many KCCs with tolerances applied it means that there are many features in the model that have to be manufactured within given limits. As a consequence the cost of production is high. That said if there are not enough KCCs in the model then it is impossible to achieve the desired level of control over the KPC and product quality cannot be assured. One advantage of the prototype process described in this work is that it doesn't require the manufacture and assembly of costly physical prototypes.

In section 4 an approach to simulate the effect of dimensional variation on a CAD model assembly was introduced. The approach used the sensitivities calculated between the CAD feature parameters and the manufactured dimensions to determine how to perturb the CAD features to simulate a given variation in a manufactured dimension. This allows the resulting variation caused by tolerance stack-up to be simulated in the CAD model assembly. This also allows for other CAD modelling analysis tools to be used for the assembly in its non-nominal state. This offers a low cost, automatable approach to tolerance analysis, compatible with 3D CAD models.

CAD assemblies are defined by multiple CAD parts whose interactions are constrained to restrict the degrees of freedom between parts. If any part geometry is modified, the assembly constraints are re-evaluated to rebuild the assembly model. The described approach requires numerous model updates when perturbing the parameters, each of which incurs an associated computational cost. The sensitivity analysis used in this work was prototyped using the Visual

Basic interface to CATIA V5. For the simple assembly models described within the update time due to perturbing an individual CAD parameter was approximately 1 second on a 2.0 GHz CPU. This could become computationally expensive where the model is complex and in situations where there are a large number of CAD parameters in the assembly model. However, the process of perturbing the parameters is highly parallelisable. Once the sensitivities are available the process of calculating the tolerances is extremely fast (in the order of seconds). It is believed these concepts could be carried out using most 3D constraint-based modellers.

This work allocates tolerances based on the worst case limits specified for the KPC dimensions. The method could also be used for statistical approaches such as Monte Carlo simulation [36], [37], [38], where different assembly dimensions could be set to different values as required and measurements taken directly from the CAD model. It was also demonstrated how the process can be used to determine if existing part tolerances in an assembly make it impossible to achieve the desired KPC dimension tolerances. Furthermore, the approach is deterministic and well-defined compared to a statistical approach that may differ in its results at different manufacturing sites [39], [40], [41] and [42].

In general, even after a parametric update the form of the CAD model remains true. Real world components do not behave in this manner and complex shape variation is to be assumed as a result of the manufacturing process. For example, corners in a CAD model are typically constrained to be 90 degrees and flat faces will remain flat throughout, both of which will exhibit small amounts of geometric variation in the real world. Also, features which nominally result in a clearance may interfere due variation caused by manufacturing processes or the external influence of internal and external loads. To correctly model such a scenario in an automated manner, it is necessary to redefine the manner in which the parts are represented in the CAD system.

6 Conclusions

The following conclusions have been drawn from this work

- The sensitivity of the change in KCC dimensions and KPC dimensions to the change in a CAD parameter can be used to predict the relationship between the KCCs and KPCs
- The relationship between KCCs and KPCs can be used to allocated dimensional limits to the KCCs based on the requirements for the tolerances on the KPCs
- The sensitivity of the change in a measured dimension to the change in a CAD parameter can be used to predict the change in the CAD parameter required to give a specified change in the measured dimension
- Predicting the parameter changes to cause given changes in the KCC dimensions provides a CAD based approach to tolerance analysis compatible with 3D CAD assemblies.

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